7-1: Exponential Functions, Growth, and Decay

goals/objectives
• Determine if an exponential function shows growth or decay
• Draw the graph of graphs of exponential functions
• Determine exponential growth or decay when given a constant percent increase or decrease.

standard 9.2.1.7

APK

Each human being has one biological father and one biological mother. Working backwards through each generation, each human (mathematically) has $2^n$ ancestors, where $n$ is the number of generations (from a single individual).

<table>
<thead>
<tr>
<th>Generations</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ancestors</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

NEW INFORMATION

What is an exponential function?

A function that shows repeated multiplication is called an exponential function.

The parent exponential function is

$$f(x) = b^x,$$

where $b > 0$ and $b \neq 1$.

What is exponential growth? What is exponential decay?

A function of the form $f(x) = ab^x$ with $a > 0$ and $b > 1$ is an exponential growth function, which increases as $x$ increases. When $0 < b < 1$, the function is called an exponential decay functions, which decreases as $x$ increases.

Note: The function $f(x) = ab^x$ is a vertical stretch/compression of the parent function.

APPLY

Tell whether the function shows growth or decay. Then graph.

How can you tell if an exponential function

Note: The graphs are on “7-1 graphs” pdf.
Example 1
\[ f(x) = 32(0.5)^x \]
Since the value of \( b \) is less than 1, this shows exponential decay.

Example 2
\[ f(x) = 0.5(1.2)^x \]
Since the value of \( b \) is greater than 1, this shows exponential growth.

Example 3
\[ f(x) = 0.4 \left( \frac{3}{4} \right)^x \]
Since the value of \( b \) is less than 1, this shows exponential decay.

Application problems

Example 1

**Biology** A acidophilus culture containing 150 bacteria doubles in population every hour. Predict the number of bacteria after 12 hours.

a. Write a function representing the bacteria population for every hour that passes.

The table representing this situation is given.

<table>
<thead>
<tr>
<th>time (hrs)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>populations</td>
<td>150</td>
<td>300</td>
<td>600</td>
<td>1200</td>
<td>2400</td>
</tr>
</tbody>
</table>

Notice that for 2 hours, the population is 150(2)(2), and for 4 hours the population is 150(2)(2)(2)(2).

The function is \( f(x) = 150(2)^x \).
How do we use exponential decay in physics?

b. Graph the function.

Graph the function on your graphing calculator. Set the window from 0 to 6 for the \( x \)-values, and from 0 to 10,000 for the \( y \)-values.

c. Use the graph to predict the number of bacteria after 12 hours.

\[ f(x) = 150(2)^x \]

\[ f(12) = 150(2)^{12} \quad \text{Substitute } x = 12. \]

After 12 hours, there will be 614,400 bacteria. (This is only an approximation, since not all bacteria will survive.)

Example 2

**Physics** A new softball dropped onto a hard surface from a height of 25 inches rebounds to about \( \frac{2}{5} \) the height on each successive bounce.

a. Write a function representing the rebound height for each bounce.

<table>
<thead>
<tr>
<th>bounce</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>height on rebound</td>
<td>25</td>
<td>10</td>
<td>4</td>
<td>( \frac{8}{5} = 1.6 )</td>
<td>( \frac{16}{25} = .64 )</td>
<td>( \frac{32}{125} = 0.256 )</td>
</tr>
</tbody>
</table>

The function for the height is \( f(x) = 25 \left( \frac{2}{5} \right)^x \).

b. Graph the function.

Graph the function on your graphing calculator. Set the window from 0 to 6 for the \( x \)-values, and from 0 to 30 for the \( y \)-values.

c. After how many bounces would a new softball rebound less than 1 inch?

4 bounces

You can model growth or decay by a constant percent increase or
How can you use exponential growth to make predictions about populations of organisms?

How do you use exponential decay to calculate depreciation?

decrease with the following formula?

\[ A(t) = a(1 \pm r)^t \]

Where \( A(t) \) is the final amount, \( a \) is the initial amount, \( r \) is the interest rate (as a decimal), and \( t \) is the number of time periods.

Example 1

**Biology** In 1981, the Australian humpback whale population was 350 and has increased at a rate of 14% each year since then. Write a function to model population growth. Use a graph to predict when the population will reach 20,000.

Since the example gives an initial starting year of 1981, we will use that as the initial time \( t = 0 \).

\[ P(t) = 350(1 + 0.14)^t \]

\[ P(t) = 350(1.14)^t \]

Use a graphing calculator to find when the population will reach 20,000. Use \( y_1 = 350 \times (1.14)^x \). Set the window for 0 to 35 for the \( x \)-values, and from 0 to 30,000 for the \( y \)-values. Graph \( y_2 = 20000 \) and find the intersection.

The population will be 20,000 whales in approximately 30.9 years.

Example 2

**Economics** A motor scooter purchased for $1000 depreciates at an annual rate of 15%. Write an exponential function, and graph the function. Use the graph to predict when the value will fall below $100.

\[ v(t) = 1000(1 - 0.15)^t \]

\[ v(t) = 1000(0.85)^t \]

Use a graphing calculator to find when the value of the scooter is $100. Use \( y_1 = 1000 \times (0.85)^x \). Set the window for 0 to 20 for the \( x \)-values, and from 0 to 1000 for the \( y \)-values. Graph \( y_2 = 100 \) and find the intersection.
The value of the scooter will be $100 in about 14.2 years.

Summary

• Know how to determine if an exponential function shows growth or decay.
• Know how to graph exponential functions on a graphing calculator.
• Know how to determine exponential growth or decay when given a constant percent increase or decrease.

Other skills needed: use of graphing calculator